Stability of stationary periodic solutions to the Euler equations

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Abstract:

Euler's equations describe the dynamics of gravity waves on the surface of an ideal fluid with arbitrary depth. In this talk, I discuss the stability of one-dimensional traveling wave solutions for the full set of Eulers equations via a generalization of a non-local formulation of the water wave problem due to Ablowitz, Fokas and Musslimani. Transforming the non-local formulation into a traveling coordinate frame, we obtain a new scalar equation for the stationary solutions using the original physical variables. Using this new equation, we develop a numerical scheme to determine traveling wave solutions by exploiting the bifurcation structure of the non-trivial periodic solutions. Next, we determine numerically the spectral stability for the periodic traveling wave solution by extending Fourier-Floquet analysis to apply to the non-local problem. We can generate the full spectra for all traveling wave solutions. In addition to recovering well-known results such as the Benjamin-Feir instability for deep water waves, we confirm the presence of high-frequency instabilities for shallow water waves. Finally, I discuss preliminary stability results of a two-dimensional surface with respect to two-dimensional perturbations.